

# *Yield-to-Maturity and the Reinvestment of Coupon Payments*

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## ABSTRACT

This note addresses a common misconception, found in investment texts and popular investment education literature, that in order to earn the yield to maturity on a coupon bond an investor must reinvest the coupon payments. We identify a sample of text and professional sources making this claim, demonstrate that yield to maturity entails no assumption of coupon reinvestment, discuss a cause for this confusion and offer a possible remedy.

## Introduction

The purpose of this note is to correct a chronic and acute error found in many investment texts and the popular investment literature. The erroneous claim is that bond holders must reinvest the coupons to “earn” the calculated yield to maturity. The reinvestment assumption claim results from confusion about how cash flows are accounted for in the calculation of the yield to maturity and the internal rate of return. It is a chronic error in that it persists in spite of continued attempts to correct it. For example, Renshaw (1957) addressed this error fifty years ago while discussing capital budgeting decisions,<sup>2</sup> but the reinvestment assumption continues to be replicated. For example, Elton, Gruber, Brown, and Goetzmann, well-respected academics and authors of a college text on portfolio theory, state “The yield to maturity is the return if all cash flows received before the horizon are invested at the yield to maturity to the horizon” and later on “In calculating the yield to maturity, the implicit assumption is that cash flows are reinvested at 6% for bond A and 6.1% for bond B (the respective yield to maturities).”(Elton, et. al. 2007, p. 507-508). A limited sampling of investment texts found the reinvestment claim restated in textbooks by Bodie, Kane and Marcus (2004), Hirt and Block (2006), and Saunders and Cornett (2007).

It is an acute problem as successive generations of financial professionals educated with the erroneous text have restated the claim in materials intended to educate investors. For example, in a series on bonds in the *American Association of Individual Investors’ Journal* Thau states, “YTM calculations are based on the assumption that coupons are never spent; they are always reinvested.” She goes on to specify that the “coupons are reinvested at an interest rate equal to the yield-to-maturity.” (Thau 2002, p. 25). In the same *Journal*, in an article entitled “The Investor Professor” states that the yield to maturity assumes “all interest payments are reinvested at a rate that is equivalent to the yield to maturity” (AAII Journal 2007, p. 15). A review of popular investment websites yielded numerous examples of investment advice that included the coupon reinvestment claim. Among the sites containing this claim are Bloomberg.com, Forexberg.com, Militarymoney.com, Investopedia.com, Morningstar.com, Netxclient.com, and even the popularly-edited Wikipedia.org.

## Yield-to-Maturity is the Return Earned

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<sup>2</sup> See Johnston et. al. for a brief note on the history of this error in capital budgeting.

The standard definition from both academic and business sources for a bond's yield-to-maturity, which also describes the method of calculation, is, "The yield-to-maturity is the single discount rate that, when applied to all future interest and principal payments, produces a present value equal to the purchase price of the security." Note there is neither compounding of interest nor a future value in the YTM calculation, and hence no assumption of coupon reinvestment. To demonstrate, suppose an investor purchases a five-year note with a 5% coupon rate at par and receives \$50 in interest annually for five years and is paid the face value at maturity. As any financial calculator will show, 5% satisfies the definition of yield-to-maturity, the discount rate that makes the present value of the coupons and face value at maturity equal to the price of the bond.

The price of the bond can be decomposed into a set of present values, each of which is the present value of a particular future cash flow. For example, the present value of \$50 in one year at 5% is \$47.62, while \$45.35 is the present value of \$50 in two years at 5% per annum. It follows that \$47.62 at 5% compounded annually becomes \$50, and \$45.35 earning 5% compounded annually for two years becomes \$50. Figure 1 illustrates the complete decomposition of discounted amount / compounded amount relationship.

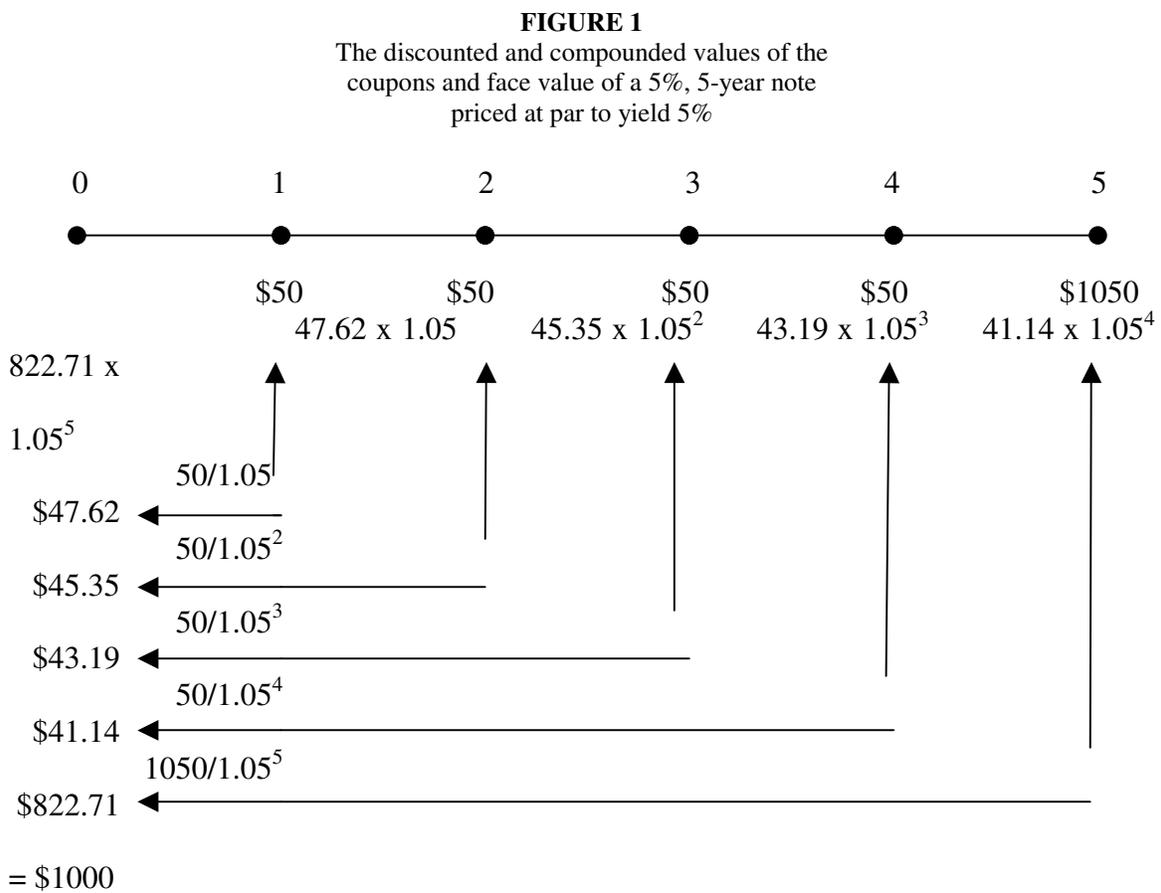


Figure 1 shows how the present value amounts that sum to the bond's \$1,000 price are *earning* a 5% compound rate of return in becoming the coupon or face value that each represents, e.g., the \$43.19 part of the \$1,000 price has earned 5% compounded for 3 years when the \$50 coupon is received in year 3 ( $\$43.19 \times 1.05^3$ ). Since each dollar of the bond's price is earning a 5% compound rate of return it follows that by paying the price of the bond and receiving the promised coupons and face value at maturity that the investor *earns* the calculated YTM.

**The Coupon Reinvestment Assumption's Persistence**

Why is the coupon reinvestment assumption so resistant to efforts to correct it? One possible explanation is contained in how finance professors, investment professionals and individual investors analyze investment results. In *"The Essays of Warren Buffett: Lessons for Corporate America"*, Buffett states, "One problem with a normal bond is that even though it pays a given interest rate — say 10% — the holder cannot be assured that a compounded 10% return will be realized. For that rate to materialize, each semi-annual coupon must be reinvested at 10% as it is received" (Buffett 2001, p. 108-109). Buffet does not claim that the YTM measure does not properly measure the earned yield but rather that it can not be used to calculate the terminal or future value of reinvested coupon payments unless the coupons are reinvested at the YTM rate. As an investor he is concerned with terminal wealth (See for instance, Hirt and Block (2006, pp. 367-368) or accumulated value. This terminal or accumulated value perspective requires knowledge of the rate earned on reinvested coupons but is a distinctly different concept than YTM. A possible remedy to the persistent incorrect use of YTM is the introduction of an additional term, perhaps "compounded yield to maturity," to represent the calculation of a terminal or future value. However, as a practical matter calculating a compounded yield to maturity is of limited use as the yield on reinvested coupons is unknowable. Indeed, calculating a compounded yield to maturity would require knowledge of the prevailing interest rate at the time of each coupon payment and each of coupons on reinvested coupons.

### Conclusion

A common qualifier to the standard YTM definition is that the "actual" or "realized" yield is subject to the coupon reinvestment at the YTM rate. With this (unnecessary) assumption the focus of yield-to-maturity is no longer on the yield the money invested in a bond will "realize" in becoming the promised payments as in Figure 1. YTM deals only with the time-value-of-money calculations between the price, coupons and face value of the bond at hand, not with other potential future investments. If the coupons and face value are paid as promised the bond earns its yield-to-maturity.

Those who claim that the yield-to-maturity *assumes* that the coupons are reinvested ascribe to it a long held but entirely different meaning. They use yield-to-maturity to describe the time-value-of-money relationship between the bond's price and the total amount of money accumulated by the bond's maturity date by reinvesting all coupons (and presumably all coupons on all coupons, etc., etc.). When discussed in this manner they are actually describing what is called the terminal wealth. But this assumption is not inherent in the meaning of YTM on *an investment* as in Figure 1. By extending the meaning of yield-to-maturity to encompass the original investment and a subsequent series of separate reinvestments, the meaning of YTM is distorted. That distortion has continued to create confusion in the financial literature.

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